5.7 – Properties of Logarithms

Daily Objectives:

1. Use the properties of exponents to multiply, divide, and exponentiate with logarithms.
2. Formally define the properties of logarithms.

Investigation: Properties of Logarithms

Step 1: Use your calculator to complete the table. Record the values to three decimal places.

Log form	Decimal form
Log 2	0.301
Log 3	.477
Log 5	.698
Log 6	.778
Log 8	.903
Log 9	.954
Log 10	1
Log 12	1.079
Log 15	1.176
Log 16	1.416
Log 25	1.397
Log 27	1.431

Step 2: Look closely at the values for the logarithms in the table. Add log2 and log3. Where can you find that sum in the table?

$.301 + .477 = .778$

Find 4 examples of pairs that add up to a third in the table. Record these equations you find the form:

$\log 2 + \log 3 = \log 6$

$\log 3 + \log 5 = \log 15$ $\quad \log 3 + \log 3 = \log 9$

$\log 3 + \log 9 = \log 27$ $\quad \log 5 + \log 5 = \log 25$

Step 3: Write a conjecture based on your results from Step 2.

$\log a + \log b = \log(ab)$

Step 4: Use your conjecture to write log(90) as the sum of two logs.

$\log 90 = \log 45 + \log 2$

$= \log 30 + \log 3$

Complete the following statement:

$\log a + \log b = \log ab$

Step 5: Now find three pairs of values in the table that subtract to equal another value in the table. Record your results in the form: $\log 9 - \log 3 = \log \_\_3\_\_$.

$\log 15 - \log 5 = \log 3$

$\log 15 - \log 3 = \log 5$

$\log 27 - \log 9 = \log 3$

What pattern do you see?

$\log a - \log b = \log \left(\frac{a}{b}\right)$

Complete the following statement:

$\log a - \log b = \log \_\_\frac{a}{b}\_\_$

Step 6: Now find values in the table that can be multiplied by a small integer to give another value in the table, such as:

$3 \cdot \log 2 = \_\_.903\_\_$. $= \log 8 \rightarrow 2^3 = 8$

You may want to think about different ways to express numbers such as 25 or 27 using exponents.

$5^2 = 25$ $2\log 5 = \log 25$ $3\log 3 = \log 27$ $2\log 9 = \log 81$

What patterns do you notice?

Complete the following statement: $b \cdot \log a = \log \_\_a^b\_\_$

Example 1: Use the properties of logarithms to rewrite each expression as a single logarithm.

a. $\log 21 - \log 7$

$\log\left(\frac{21}{7}\right)$

$\log 3$

b. $-4\log 2$

$\log 2^{-4}$

$\log\left(\frac{1}{2^4}\right)$

$\log\left(\frac{1}{16}\right)$

c. $-2\log 5 + 4\log 5$

$\log 5^{-2} + \log 5^4$

$\log\left(5^{-2} \cdot 5^4\right)$

$\log\left(5^2\right)$

$\log 25$

Example 2: Write each expression as a sum or difference of logarithms. Simplify the result if possible.

a. $\log_5 \dfrac{a\sqrt{b}}{c^4}$

$\log_5 a + \log_5 b^{\frac{1}{2}} - \log_5 c$

$\log_5 a + \frac{1}{2}\log_5 b - \log_5 c$

b. $\log_4\left(\sqrt{r}\cdot\sqrt[3]{s}\cdot\sqrt[4]{t^3}\right)$

$\log_4 r^{\frac{1}{2}} + \log_4 s^{\frac{1}{3}} + \log_4 t^{\frac{3}{4}}$

$\frac{1}{2}\log_4 r + \frac{1}{3}\log_4 s + \frac{3}{4}\log_4 t$

Example 3: Determine whether each equation is true or false.

a. $\log 8 = \dfrac{\log 32}{\log 4}$

FALSE

$\log 8 = \log\left(\frac{32}{4}\right)$

b. $\log\sqrt{6} = -2\log 6$

$\log 6^{\frac{1}{2}} =$

$\frac{1}{2}\log 6 \neq -2\log 6$

FALSE

$\log\sqrt{6} = \frac{1}{2}\log 6$